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GEOGEBRA ASSISTED SCAFFOLDING: STRATEGIES TO SUPPORT STUDENT GEOMETRY PROBLEM-SOLVING ABILITY

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Abstrak

Aktivitas penyelesaian masalah merupakan aktifitas yang dapat digunakan untuk melihat kemampuan matematis siswa. Teknologi seperti komputer yang berkembang pesat dapat menjadi sumber alternatif untuk membantu proses pembelajaran. Dengan demikian penggunaan teknologi dalam scaffolding merupakan hal yang dapat dipertimbangkan. Khalil et al (2019) menyatakan bahwa terdapat pengaruh positif penggunaan geogebra sebagai alat scaffolding untuk penyelesaian geometri. Secara khusus, masih perlu dilakukan penelitian lebih lanjut yang mengungkap bagaimana penerapan Scaffolding berbantuan GeoGebra dapat mendukung kemampuan pemecahan masalah geometri siswa. Penelitian ini merupakan penelitian deskriptif dengan pendekatan kualitatif. Subjek penelitian adalah mahasiswa Program Studi Pendidikan Matematika semester empat yang berjumlah tiga orang dengan kemampuan geometri tinggi, sedang, rendah. Metode penelitian terdiri dari tiga tahapan yaitu 1) studi pendahuluan, 2) perencanaan, dan 3) implementasi. Teknik analisis data yang digunakan terdiri dari tiga tahapan yaitu 1) reduksi data, 2) penyajian data, 3) penarikan kesimpulan. Hasil penelitian menunjukkan bahwa scaffolding berbantuan geogebra diberikan dalam bentuk *explaining* pada subjek sedang dan pada subjek rendah dalam bentuk *explaining* dan *reviewing* pada fase analisis serta dalam bentuk *restructuring* pada fase perencanaan. Hasil penelitian ini dapat digunakan sebagai acuan guru yang memanfaatkan aplikasi Geogebra dalam rangka memberikan scaffolding penyelesaian masalah geometri sesuai dengan kemampuan matematika siswa. Sehingga bentuk scaffolding berbasis geogebra yang diberikan dapat membantu meningkatkan kemampuan matematis siswa secara maksimal.

Abstract

Problem-solving activity is an activity that can be used to see students' mathematical abilities. Technology such as computers that are developing rapidly can be an alternative source to assist the learning process. The use of technology in Scaffolding is something that can be considered. Khalil et al (2019) stated that using GeoGebra as a scaffolding tool for solving geometry has a positive influence. In particular, further research must be carried out to reveal how applying GeoGebra-assisted scaffolding can support students' geometric problem-solving abilities. This research is descriptive research with a qualitative approach. The research subjects were students of the Mathematics Education Study Program in the fourth semester, totaling three people with high, medium, and low geometric abilities. The research method consists of three stages: 1) preliminary study, 2) planning, and 3) implementation. The data analysis technique consisted of three stages, namely 1) data reduction, 2) data presentation, and 3) concluding. The results showed that GeoGebra-assisted Scaffolding was given in the form of explaining for moderate subjects and for low-level subjects in the form of explaining and reviewing in the analysis phase and in the form of restructuring in the planning phase. The research results can be used as a reference for teachers who use the Geogebra application to provide scaffolding for solving geometric problems according to students' mathematical abilities. So that the form of geogebra-based scaffolding provided can help improve students' mathematical abilities to the maximum.

Keywords*: GeoGebra, Geometry, Problem Solving, Scaffolding*

PENDAHULUAN

Problem-solving activity is an activity that can be used to see students' mathematical abilities. Problem-solving is essential to learning mathematics (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016). One of the reasons is that problem-solving skills can help students to learn to think mathematically (Rott, 2021). In particular, the core of the goal of learning mathematics is that students have skills in solving problems by utilizing the mathematical concepts they have learned. According to Szabo et al

(2020), teaching mathematical problem-solving is an effective way to deal with various problems that develop. Rott (2021) states that there are five problem-solving indicators: analysis, exploration (exploring aspects of the problem), planning, implementation, and verification. The right strategy is needed to support students' problem-solving abilities. Using appropriate learning strategies is one of the factors that can influence students' success in becoming successful problem solvers (Schoenfeld, 2016).

The scaffolding strategy was introduced by Wood, Brunner, and Ross (1976) as a gradual guidance or provision of assistance to enable a child to solve a problem. According to Makar et al (2015), Scaffolding is temporary support provided by a teacher or knowledgeable person to help students solve problems they cannot solve independently. This support can be offered in different ways, such as modeling, problem posing, and others (Dukuzumuremyi & Siklander, 2018; Muhonen, Rasku-Puttonen, Pakarinen, Poikkeus, & Lerkkanen, 2016). Scaffolding has received much attention as an effective learning strategy because it helps students engage in learning and improves learning outcomes (Belland, 2014). Research on Scaffolding generally focuses on one-on-one or small-group guidance. Teachers play an essential role in the scaffolding process by providing adaptive support that can enhance student learning (Chen & Wu, 2016). Technology such as proliferating computers can be an alternative source to assist the learning process. So the use of technology in Scaffolding is something that can be considered. According to Belland (2014); Kim & Hannafin (2011) as scaffolding technology develops, it depends not only on the scaffolding provider but also on technology, such as the software used. GeoGebra is a software that offers interactive learning that allows students to explore various mathematical concepts (Jelatu, Sariyasa, & Made Ardana, 2018). GeoGebra can help teachers increase students' understanding of mathematical concepts and procedures because this software offers mathematical functions such as symbols, graphs, and many others (Zulnaidi & Zamri, 2017). There have been many studies linking GeoGebra with learning, for example, in stereometry learning (Kramarenko, Pylypenko, & Muzyka, 2020), proof (Botana et al., 2015) conceptual understanding of integral material (Tatar & Zengin, 2016) as well as geometry (Orozco & Morales-Morgado, 2017; Venema, 2011) curriculum, geometry is one of the essential materials taught from elementary school to university. This is because geometry helps us understand the world by comparing shapes, objects, and relationships (Gunhan, 2014). Understanding geometry is essential for understanding other areas of mathematics. This contributes to logical and deductive reasoning about objects and spatial relationships (Alqahtani & Powell, 2016). Therefore, understanding the concept of geometry must be developed effectively in learning mathematics. Susilawati et al (2021) explained that teaching geometry is expected to be able to provide an attitude of visualizing the relationships and characteristics of geometric spaces. Susilawati et al (2021) emphasizes that students must be given opportunities and adequate supporting learning media to observe, explore, and find geometric principles through informal activities before informing what they have learned in their formal learning activities.

Khalil et al (2019) conducted research related to the use of GeoGebra as a scaffolding medium on geometric material. The main focus of this research is to see the effect of geo on students' mathematical thinking using a quantitative approach. In particular, there still needs to be more research that reveals how applying GeoGebra-assisted Scaffolding can support students' geometry problem-solving abilities. This research is essential, considering problem-solving relates to various mathematical activities such as representation and reflective thinking (Dörner & Funke, 2017). Based on some of these descriptions, the formulation of the problem that the researcher will raise is how to apply GeoGebra-assisted Scaffolding to support geometric problem-solving abilities. The research aims to describe the application of GeoGebra-assisted Scaffolding, which can support the ability to solve geometric problems.

RESEARCH METHOD

This research is descriptive research with a qualitative approach. The qualitative approach was chosen because the research aims to describe the application of GeoGebra-assisted Scaffolding, which can support the ability to solve geometric problems. The research subjects were students of the Mathematics Education Study Program in the fourth semester. Three students were selected randomly according to criteria with the evaluation scores of geometry in the previous chapter being high, medium, and low—determination of subject criteria based on the evaluation value obtained on the material before geometric flat wakes. Apart from being based on scores, another criterion is that the subject can complete the tasks given seriously. This is done to obtain data relating to the shape of the GeoGebra rock scaffolding that will be provided. Next, each subject was given a geometry task to solve. Before being given to students, the geometry assignment was first assessed in terms of content and language. The content aspect assesses the suitability of the assignment to the educational level, and the language aspect assesses readability. The research instrument used was observation sheets of student and lecturer activities. Observation sheets are used to obtain qualitative data related to student problem-solving activities and Scaffolding carried out by lecturers. Apart from using observation sheets, solving problems and providing scaffolding was recorded using an audio recorder. So that researchers can increase their diligence to ensure the validity of research data. Sugiyono (2019) stated that the validity of research data can be achieved by increasing the researcher's perseverance in reading the data obtained. The supporting instrument is GeoGebra software.

The research method consists of three stages: 1) preliminary study, 2) planning, and 3) implementation. The preliminary study consists of examining scaffolding theory, problem-solving, and GeoGebra. The planning stage involves making research instruments, namely observation sheets of student and lecturer activities and the selection of subjects. Implementation is the stage of collecting data, analyzing data, and drawing conclusions. Data collection is carried out at the implementation stage by providing geometry-related problems on the subject determined at the planning stage. The results of the subject's work were analyzed to determine the shape of the Scaffolding assisted by the given GeoGebra. The scaffolding shape is given through the zoom platform individually so that it is detected in detail the scaffolding shape needed between high, medium, and low subjects in solving geometry problems. In this case, the instrument used is the observation sheet. After the observation data is obtained, data analysis and conclusions are drawn.

The data analysis technique consists of three stages, namely 1) data reduction, 2) data presentation, and 3) concluding. Data reduction is choosing primary data, namely choosing data that meets all indicators of problem-solving activities so that the form of GeoGebra-assisted Scaffolding can be identified at each step of problem-solving. Furthermore, data related to Scaffolding to support problem-solving abilities that have been analyzed are presented as a description. And finally, drawing conclusions based on the results of the description. The problem-solving indicators refer to the research results Rott (2021) of analysis, exploration, planning, implementation, and verification. *Analysis* is discuss the core of the problem, the process of solving it, and the analogy with similar problems; *Exploration* is discuss in an unstructured manner the core of the problem, and the process of solving it, analogizing the problem with similar problems; *Planning* is adopt known procedures. We are combining known procedures in the context of a new problem; *Implementation* is carry out activities contained in the planning phase; *Verification* is check results by looking back at recently resolved issues.

While the form of Scaffolding given refers to Anghileri (2006) consists of three components, namely explaining, reviewing, and restructuring. Activities in explaining include telling and showing, namely the preparation of initial discussion material involving a little student contribution. Reviewing is an activity to refocus on the core issues. This activity anticipates if students need help to identify the essential aspects of a concept or problem. The restructuring component introduces modifications of emerging ideas so that they produce students' understanding and the ability to interpret a concept.

RESULTS AND DISCUSSION

The preliminary study consists of examining scaffolding theory, problem-solving, and GeoGebra. The results of the examination showed that not all students were able to solve the problems given. Thus, scaffolding is needed to help students solve a problem. Technology such as computers, which are developing rapidly, can be an alternative source. The Geogebra application can be used as an alternative. Quantitative research results also show that Geogebra rock scaffolding positively affects students' geometric problem-solving abilities. The planning stage involves making research instruments, namely observation sheets of student and lecturer activities and the selection of subjects. Implementation is the stage of collecting data, analyzing

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data, and drawing conclusions. Data collection is carried out at the implementation stage by providing geometry-related problems on the subject determined at the planning stage. The results of the subject's work were analyzed to determine the shape of the Scaffolding assisted by the given GeoGebra. The scaffolding shape is given through the zoom platform individually so that it is detected in detail the scaffolding shape needed between high, medium, and low subjects in solving geometry problems. The following are the research results on applying GeoGebra-assisted Scaffolding to support problem-solving abilities in the geometry material given to subjects with high, medium, and low criteria. The problems given by researchers are as follows.

"Given the cube ABCDEFGH with a side length of 2 units. If points K and L respectively are the midpoints of the lines BF and DH, while EC is the space diagonal of the cube, then determine the distance of point G to the KCLE plane".

High Geometry Ability Subject

Based on the results of observations in solving problems given by subjects with high criteria fulfilling all problem-solving indicators, lecturers do not carry out Scaffolding. The following details describe the stages of solving high-subject problems. In the analysis stage, the subject can determine the essence of the given problem. This can be seen in Figure 2, which shows that the subject can correctly determine the line's location where the distance from point G to the KCLE plane is on the CE line.

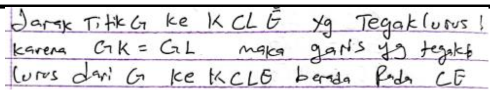

Figure 2. High Geometry Ability Subject Analysis Phases

Next, the subject completes the exploration phase by determining which lines can help solve the problem. In this case, the subject first determines the distance from point G to point E, from point G to point K, and from point G to point L. From the exploration and analysis phase, the subject who previously only determined the location of the distance after exploring can determine the line representing the distance of point G with the KCLE field, i.e., the GM line. This can be seen in Figure 3 below.

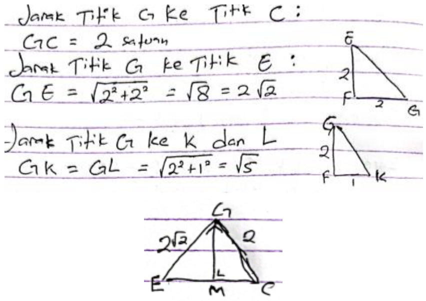

Figure 3. High Geometry Ability Subject Exploration Phase

Based on the results of the exploration phase, the subject makes a problem-solving plan using a comparison of the areas of triangles. This can be seen from the subject's answers at the stages of calculating the area of the EGC right triangle, as shown in Figure 4 below.

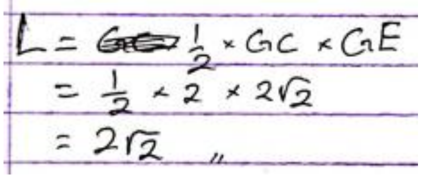

Figure 4. Planning Phase of High Geometry Ability Subject

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The implementation phase can be seen from the tall subject's answers using a comparison of the area of the CGE triangle. The first area comparison uses the GE and GC line segments as the base and height, and the second area uses the EC and GM line segments as the base and height. As seen in Figure 5 below, by comparing the areas of the triangles, the length of the line segment GM is obtained, which is the distance from point G to the KCLE plane.

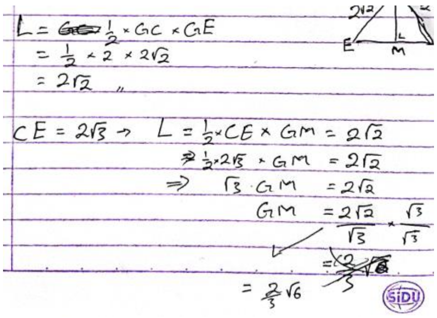
$L = \frac{1}{2} \times GC \times GE$
$= \frac{1}{2} \times 2 \times 2\sqrt{2}$
$= 2\sqrt{2}$
$CE = 2\sqrt{3} \rightarrow L = \frac{1}{2} \times CE \times GM = 2\sqrt{2}$
$\Rightarrow \frac{1}{2} \cdot 2\sqrt{3} \times GM = 2\sqrt{2}$
$\Rightarrow \sqrt{3} \cdot GM = 2\sqrt{2}$
$GM = \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{2}{3}\sqrt{6}$

Figure 5. Planning Phase of High Geometry Ability Subject

Furthermore, at the verification stage, the subject re-examined the problem-solving process. An examination can be seen from the conclusions written by the subject. In this phase, the subject also checks the suitability between the problems given and the final results obtained. It can be seen from the subject that the closest distance from point G to the KCLE plane is a perpendicular line from point G to the KCLE plane. This can be seen in Figure 6 below.

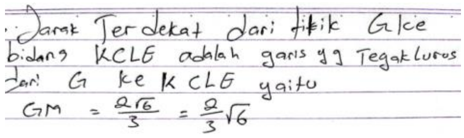
Jarak Terdekat dari titik G ke bidang KCLE adalah garis yg Tegak lurus dari G ke KCLE yaitu
$GM = \frac{2\sqrt{6}}{3} = \frac{2}{3}\sqrt{6}$

Figure 6. Verification Phase of High Geometry Ability Subjects

Moderate Geometry Ability Subject

Based on the results of observations, the following describes the stages of problem-solving and the form of Scaffolding given in solving a given problem. In the analysis phase, the subject can understand the essence of the given problem. This can be seen in drawing spatial shapes to help visualize the given problems, as shown in Figure 7 below. In the process of drawing the geometric shapes, it can be seen that the subject added a dotted line to the EC line segment, which indicates the subject's guess about the location of the line segment, which is the distance between point G and the KCLE plane. At this stage, there is no scaffolding provided by the lecturer.

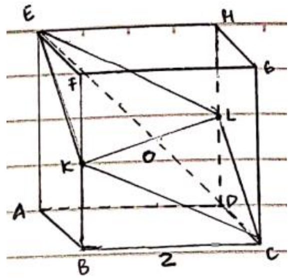

Figure 7. Moderate Geometry Ability Subject Analysis Phase

The subject's exploration phase is shown by determining the distance from point G to each corner point of the KCLE plane, namely from point G to points K, L, C, and E. Meanwhile, the planning phase is seen when the subject determines point O where this point O is the point of intersection of line segments KL and EC, and creates a line segment GO. Furthermore, the implementation phase is shown when the subject determines the distance from point G to point O. Furthermore, the verification phase is shown when the subject concludes that the distance from point G to the KCLE plane is the distance

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between point G to point O, which is equal to √1. Figure 8 below shows the stages of solving problems from a medium subject.

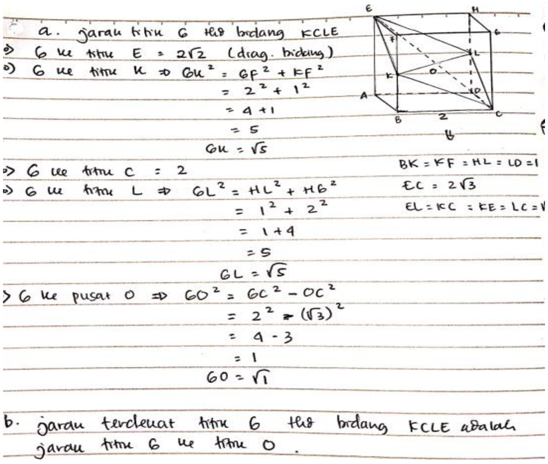

Figure 8. Stages of Problem-Solving Subject Moderate Geometry Ability

Based on the problem-solving process, the lecturer provided Scaffolding at the planning stage. The form of giving Scaffolding is in the form of explaining, in which the subject is asked to define the definition of distance. The result of Scaffolding in explaining this subject shows doubts about the answer. This can be seen when the subject, with the help of the GeoGebra application, connects point G to point E to form an ECG polygon. Then remove the polygon in a two-dimensional type so that it forms an ECG triangle, as shown in Figure 9 below.

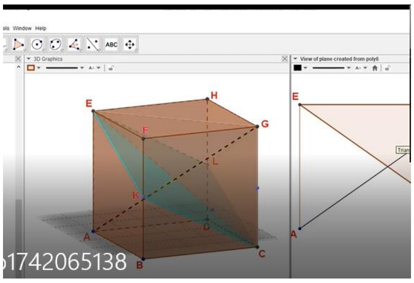

Figure 9. Scaffolding Explaining at the Planning Stage of the Moderate Geometry Ability Subject

Based on the results of the Scaffolding, the subject then changed the planning phase so that it was successful in the implementation and verification phases. The success of the implementation and verification phases is shown in Figure 10 below.

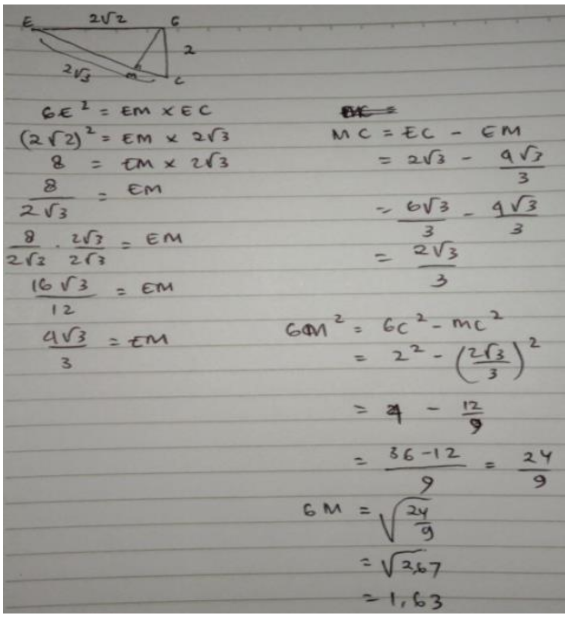

Figure 10. Stages of Problem-Solving Subject Moderate Geometry Ability After Being Given Scaffolding Explaining

As seen in Figure 10, based on the GeoGebra application, the subject determines the distance from point G to the KCLE plane by drawing a CGE triangle. It determines the distance from point G to the KCLE plane, namely the line segment GM where GM is a line perpendicular to the line segment CE. Furthermore, in the implementation phase, the subject determines the length of a line segment GM using triangle congruence and the Pythagorean theorem. The subject carried out the verification phase with the help of the GeoGebra application, namely comparing the distance from point G to all points on the CE line segment.

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Low Geometry Ability Subject

Based on the results of observations, low subjects experienced difficulties at the analysis stage. The difficulty experienced was that the subject could not visualize the given geometric problem. This impacts the subject's ability to determine the essence of the given problem. This can be seen from the subject's answers, as shown in Figure 11 below.

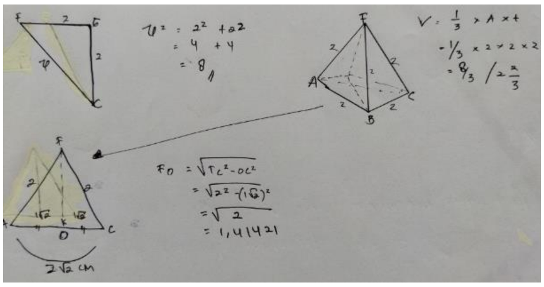

Figure 11. Stages of Problem-Solving Low Geometry Ability Subject

In Figure 11 above, it can be seen that the subject first drew the FCG triangle. Then determine the side lengths of the line segments FG and GC with a length of two units. Next, the subject determines the length of the line segment x using the formula x^2=2^2+2^2 and finds x^2=8. Only after this stage will the subject realize that the line segment is x. This stage is necessary for the subject to connect the first stage with the second completion stage. This can be seen from the second image made by the subject, namely the F.ABCD pyramid. The x value obtained (although it has yet to obtain the actual x value) is not used in the pyramid drawing. The pyramid image also shows that the subject cannot visualize the given geometric problem where the length of all the sides of the pyramid made is 2 units. The subject also determines the volume of the pyramid. The results of the pyramid volume are also unrelated to the next step, namely determining the FO length in the third triangle that is made, namely the AFC triangle. Based on the irregularity of the problem-solving stages that the subject did, it can be concluded that the subject experienced difficulties in the analysis phase, namely determining the essence of the given problem.

Based on these difficulties, the lecturer then gave Scaffolding in the form of explaining. With the help of GeoGebra, the lecturer explains the concept of point-to-point distance, point-to-line distance, and point-to-plane distance. For the point-to-point distance, the lecturer places two points, namely point A and point B, in a Cartesian plane. Then the lecturer shifts point B in several coordinates until the subject finds the concept of distance, namely the shortest path. Then the subject uses the concept of distance to determine the distance of a point with a line. At this stage, the subject found a relationship between the perpendicularity and the concept of distance that the subject had found before. Furthermore, Scaffolding in the form of reviewing was given when the subject was asked to determine the distance from a point to the plane, namely, determining the distance from point G to the plane ABCD. The results of the review can be seen in Figure 12 below.

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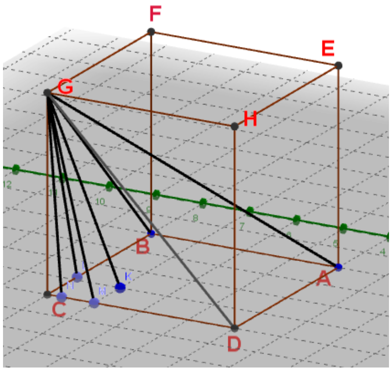


Figure 12. Results of Reviewing Subjects of Low Geometry Ability in GeoGebra

In Figure 12, it can be seen that the subject made several line segments to help find the line segment, which represents the distance between point G to the plane ABCD. Using the help of GeoGebra, intuitively, the low subject can determine the distance between point G to the plane ABCD, namely the line segment GC.

After the subject understands the concept of distance, the subject returns to the given problem, and the subject successfully completes the analysis phase. Demonstrated with the subject can determine the essence of the problem given. The exploration phase was also successfully carried out, as shown by the subject's ability to determine several line segments that could assist in solving the problem, namely the GE and EC line segments. In addition, it is shown by the subject drawing the EGC triangle to make it easier to determine the line segment representing the distance from point G to the KCLE plane. Some of the above can be seen in Figure 13 below.

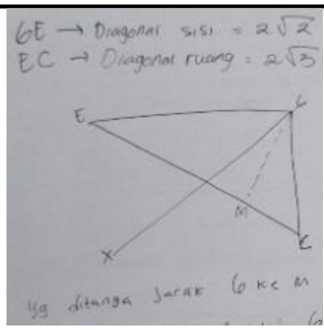
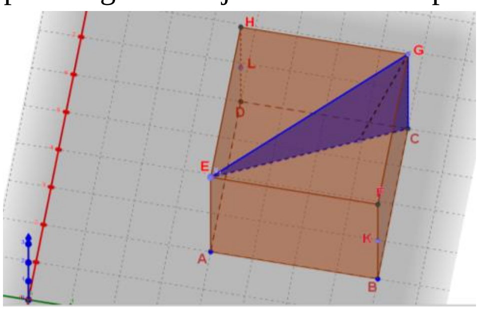


Figure 13. Analysis and Exploration Phase of Subjects with Low Geometry Ability After being given Scaffolding, Explaining and Reviewing

In the planning phase, the subject again experienced difficulties in determining the length of the GM line segment, representing the distance between point G and the KCLE plane. This can be seen when the subject says that the subject has no idea how to determine the length of a line segment GM. To overcome this, the lecturer carried out Scaffolding in the form of restructuring, namely modifying the triangles the GeoGebra assistant subject made. The modification is done by showing the position of the triangle on the cube, then rotating the cube so that you can see the base and height of the triangle from two different perspectives. The first angle of view obtains the result of the GE base and GC height, and the second point of view is the EC base and GM height. Figure 14 below shows the result of restructuring using the GeoGebra application, which affects the planning and subject verification phases.



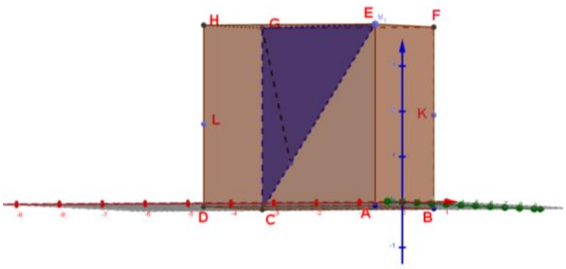


Figure 14. Results of Restructuring Low Geometry Ability Subjects in GeoGebra

Based on the research results above, GeoGebra-assisted Scaffolding supports problem-solving abilities in geometric material. GeoGebra-assisted Scaffolding helps the process of reasoning abductively, inductively, and deductively in solving problems (Uygan & Bozkurt, 2019). This reasoning is always used in every phase of problem-solving. In addition, in linking one phase to another, abductive reasoning is required (Marasabessy & Hasanah, 2021). In subjects with moderate abilities, geoge-bra-assisted Scaffolding can help subjects in the planning phase. Through the provision of Scaffolding in the form of explaining, doubts arise in the subject about the answer. The subject then verifies these doubts by removing the field corresponding to the problem so that it is visible. Using GeoGebra makes this easy and produces precise field sizes so that the subject can see the location of the error and correct it, which impacts the success of the next problem-solving phase. In line with the opinion of Bwalya (2019), who states that geo-geography can help demonstrate and visualize so that it can help solve problems appropriately.

Applying GeoGebra-assisted Scaffolding is also an excellent strategy to support problem-solving in the analysis phase. The analysis phase is a foothold in the problem-solving stage as the basis for the next problem-solving stage. The analysis phase is where the subject finds the core of the problem, the process of solving it, and analogies the problem with similar problems. Arriving at this stage requires a solid conceptual understanding so that the analysis phase can be carried out appropriately. GeoGebra-assisted Scaffolding is proven to help the subject overcome the difficulties the subject is facing, namely finding the concept of distance. Through GeoGebra, the subject can explore what is contained in his intuition so that he gets the right concept that can be used to solve problems. This is by research (Jelatu, Sariyasa;, et al., 2018) which states that GeoGebra scaffolding can help understand procedural and conceptual concepts.

KESIMPULAN

GeoGebra-assisted Scaffolding is proven to be used to support geometry problem-solving abilities. GeoGebra-assisted Scaffolding on high subjects is unnecessary. On low subjects, GeoGebra-assisted Scaffolding is carried out in the planning phase by explaining; that is, the subject removes the field from the cube to convince his intuition to get the right problem-solving plan. On low subjects, GeoGebra-assisted Scaffolding was carried out in the analysis and planning phases. The analysis phase is given Scaffolding in the form of explaining and reviewing by visualizing the concept of distance in GeoGebra. The planning phase is given Scaffolding in the form of restructuring, namely by modifying the subject's perspective with the help of the GeoGebra application. The research results can be used as a reference for teachers who use the Geogebra application to provide scaffolding for solving geometric problems according to students' mathematical abilities. So that the form of geogebra-based scaffolding provided can help improve students' mathematical abilities to the maximum.

DAFTAR PUSTAKA

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