

Analyzing Elementary School Students' Geometry Comprehension Based on Van Hiel's Theory

Chairil Faif¹

chairil@gmail.com¹

Universitas Lambung Mangkurat¹

Abstract:

Teaching geometry in elementary school sets out to build and develop students' logical thinking skills. Comprehension of geometry topics is expected to develop the students' skills to solve everyday problems. However, this learning objective has not been met as expected. In Indonesia, there are many elementary school students who still do not understand the basic concepts of geometry, such as shapes and planes. Based on this problem, the researcher conducted a study to examine students' comprehension of geometric concepts. This study is of an experimental kind and involves two classes: an experimental class and a control class. To solve the problem of learning geometric topics, the researcher utilizes Van Hiel's theory. Geometry learning in the first class is delivered using a learning approach based on the Van-Hielé theory (experimental group), while learning in the second class is delivered with the conventional approach (control group). This study comprised 150 students, drawn from primary five, representing two classifications of medium and low performance from the selected schools found in Banjarmasin. Before conducting the study, the researcher designed a learning scenario and research procedure to ensure the study went as expected. This study concludes that students' comprehension of geometric concepts after Van-Hielé's Theory-Based Geometry Learning belongs to the medium category, which is better than the students' comprehension of geometric concepts after conventional learning, which is in the low category. Conclusively, Van Hiel's geometry learning theory is found to be effective in teaching and learning mathematics.

Keywords: Concept comprehension, Elementary school, learning geometry, mathematics communication, Van Hiel's learning theory

Introduction

Geometry is a fundamental component in elementary mathematics school curriculum [31]. Teaching and learning geometry can help to develop and improve students' logical thinking skills (Sheen, 2017; Phillips, 2017; Nicholas, 2019). This is in line with Kennedy, 2019 who states that "learning experience attained through geometry learning leads to the development of problem-solving skills, reasoning skills and comprehension of other mathematical concepts." Many mathematical concepts can be explained through geometric representation. In addition, there are many geometric shapes that can be found in daily life, such as the shapes of houses, doors, blackboards, tiles, et cetera; so that these geometric shapes are familiar to primary school students. This short outline is enough to explain why geometry is important to be appropriately studied by primary school students [30].

Vojkuvkova (2020) presents views on geometric thinking in schools, concerning the theory of geometry learning. There are two points obtained from the Van Hiele theory, they are: level of thinking and learning stages [8]. There are five hierarchical stages of students' learning of geometric concepts, which have been developed or designed based on age (Bitchener, 2020). These levels have varying indicators. From the Van Hiele theory, learning concept, there are also five sequential phases that are useful in learning geometry: required data, guided orientation, interpreting meaning, open-ended orientation, merging and integration. Chang (2019) explain in details characteristics of the five levels of van Hiele's geometric thinking concepts.

Based on these characteristics, to avoid difficulties in students' geometric learning, teachers and authors (in teaching or writing textbooks on van Hiele-based geometry learning) should employ communication technique and language that are compatible with students' level of thinking.

Review of Literature

Several studies on geometry (Sheen, 2017; Phillips, 2017; Nicholas, 2019) report that many primary school students still have difficulties and make errors in comprehending geometric concepts. For instance, students think that a shape with four sides is a square, and that only an equilateral triangle is a triangle (Sheen, 2017; Phillips, 2017; Nicholas, 2019); many primary school students fail to comprehend the basic concepts of geometry and most fifth grade students erroneously think that a rectangle is a square and that a

triangle is right-angled triangle Chang (2020). Other studies that employ geometry learning based on the van Hiele theory find that fifth grade students have difficulties in understanding the characteristics of the rectangle; that learning using Van Hiele's levels can solve students' difficulties in understanding geometric shapes ; that primary school students still have difficulties in identifying the rectangle (Sheen, 2017; Phillips, 2017; Nicholas, 2019); that students' geometric comprehension positively correlates to the achievement in their study of geometric shapes; and that students' achievement and motivation in geometry improve when using the van Hiele theory.

Learning using Van Hiele's phases of understanding leads to change in students' perception. It is therefore concluded that Van Hiele's geometry thinking concepts can help to improve students' achievement and motivation in learning mathematics.

Method

This study is an experimental study, using the design of pre-test post-test control group with Van Hiele-based geometry learning approach. The study uses two different classes, the experiment group and the control group. In the former, Van Hiele-Based Geometry Learning is implemented, while in the latter, conventional geometry learning is delivered.

This study involved 150 students, drawn from primary five, representing two classifications of the medium and low performances from the selected schools found in Banjarmasin. The sampling process was conducted by considering the Banjarmasin City Education Department's categorization of schools: high level, medium level, and low level schools.

The learning scenario used in this study is a series of written learning materials that serves to guide the researcher in implementing Van Hiele-Based Geometry Learning. The research instruments used in this study is Van Hiele Geometry Test, i.e. pre-test and post-test of concept comprehension skills developed by the researcher.

This study begins with preparing everything needed to support the implementation of the study. This preparation stage includes a literary study of geometry concepts comprehension and Van Hiele-based geometry learning, and designing the implementation of Van Hiele-based geometry learning. When the instruments are ready, the next step is sampling, followed by administering pre-test. The final step is administering the post-test.

The tool used in processing the data is the software of Microsoft Office Excel 2007 and SPSS V.16. The data analysis was conducted using t-test, Two-Way ANOVA, and Scheffe test, after normality and homogeneity tests. The development and validation of instruments' characteristics follow. The following is an illustration of the test item used:

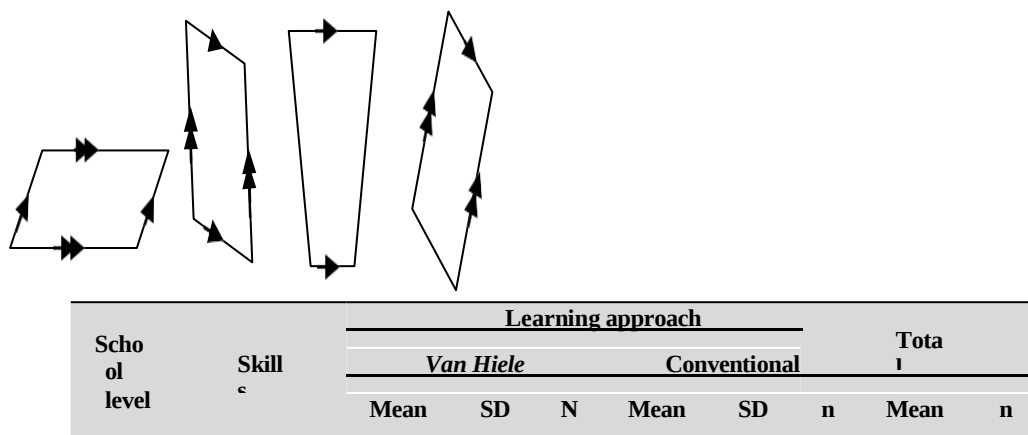


Figure 1. Examples of geometry concept comprehension test item

The data were analyzed using a two-way ANOVA test (mean difference and existence of interaction between variables), Scheffe test (mean difference test), and chi-square test (association test)

Results and Discussions

A. Analysis of Students' Comprehension of Geometry Concepts

The data analysis in this stage consists of the variance test of the experiment class (class that learns through Van Hiele-Based Geometry Learning/VH) and the control class (class that learns through Conventional Geometry Learning/Kv), normality test, and homogeneity test.

To measure the variance of the experiment and control classes, an analysis of students' scores of preliminary test of conceptual comprehension and Mathematics communication skills is conducted, based on the geometry learning approach and school level. Normality test is conducted using Kolmogorov-Smirnov Test (KS), while the homogeneity test is conducted using F-Test. The significance of mean deviance is tested using t-Test.

The preliminary test finds that students' conceptual comprehension and Mathematics communication skills before the treatment are in the Low category for the high, medium, and low levels of primary school. Table 1 displays the description of students' preliminary conceptual comprehension based on school level and student group which received training.

Overall, and in each school level, the results of pre-test on students' conceptual comprehension in class using van Hiele-based geometry learning (VH class) and in class that employs conventional learning (Cv class) are relatively low and are not significantly different. However, after treatment, the overall achievement of VH students is on sufficient category (32.59 of 48) and better than students' conceptual comprehension in Cv class (22.67 of 48). Therefore, from normalized gain, VH class students achieve higher gain (0.51) than Cv students' gain (0.20).

At each school level, in both classes (VH and Cv), students at VH class of low level and medium level schools achieved and gained higher conceptual comprehension skill than students at Cv class of high level school. This finding indicates that Van Hiele-based geometry learning plays greater role than conventional learning and school level in achieving conceptual comprehension skill.

The results of both tests on learners' conceptual comprehension in VH and Cv classes are presented in Figure 2.

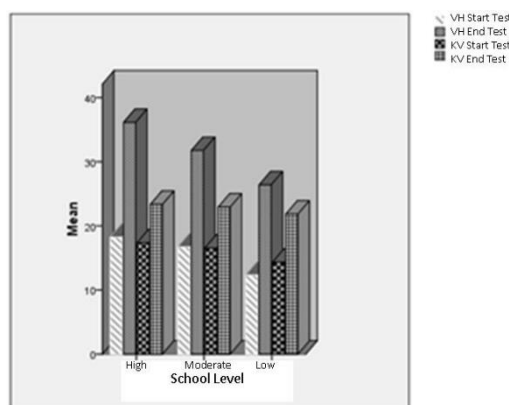


Figure 2. Conceptual Comprehension based on School Level and Learning Approach

From geometry thinking level, some students in VH class achieve the thinking level of high visualization, some students achieve the level of low analysis, and some other achieve low level of informal deduction. Meanwhile, students at Cv class only achieve low visualization level.

Therefore, Van Hiele-based geometry learning is better than conventional learning in improving the level of students' geometry thinking.

Based on the data from Table 1, the following findings are revealed: in general and on each school level, students' results regarding the test showed that both VH and Cv classes are in the category of low and are not significantly different. However, after the treatment, the students in VH class in general achieve good mathematical communication skill (27.07 of 32); which is better than mathematical communication skill of students in Cv class (14.52 of 32). From normalized gain, VH students have greater gain (0.78) than students in CV class (0.26).

At each school level, in both classes (VH and Cv), mathematical communication skills appeared to be higher. However, students at VH class of low level and medium level schools achieved and gained higher mathematical communication skill than students at Cv class of high level school. This finding indicates that Van Hiele-based geometry learning plays greater role than conventional learning and school level in achieving skill and attaining mathematical communication gain.

The result from both tests showed improved mathematical communication skills in the experimental and low in the control (conventional) class.

Ideal score: 32

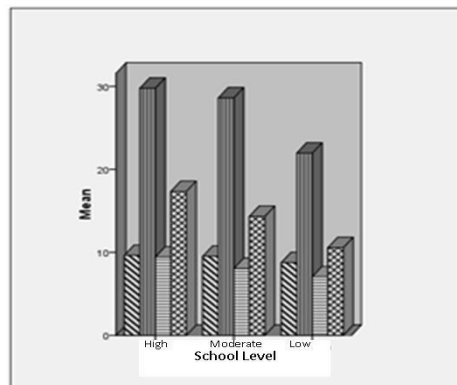


Figure 3. Students' Mathematical Communication based on School Level and Learning Approach

B. Analysis of Final Geometry Concepts Comprehension Skill

The analysis of students' final comprehension of geometry concepts (post-treatment) begins with examining students' final comprehension of geometry concepts based on the learning approaches and the school levels. To find out whether the learning approaches and school levels provide significant effect on students' comprehension, the Two-Way ANOVA test is used. The result of this test is presented in Table 4.

Class that will be treated with learning approach	N	KS	p	Normality
VH	83	1.071	0.201	Normal
Kv	75	0.808	0.532	Normal

Table 4.
Summary of Two-way ANOVA Test on Students' Final Geometry Concepts Comprehension based on School Levels and Learning Approaches

Source	JK	Dk	RJK	F _{calc}	F _{critical}
School levels (A)	603.019	2	301.509	6.703	
Geometry learning approaches (B)	3748.987	1	3748.987	83.346	3.05
A x B	207.284	2	103.642	2.304	

C. Learning Approach

To analyze the data in Table 4, the following criteria are used.

$H_0: \mu_e = \mu_k$ (there is no difference between students' final geometry concepts comprehension in experimental group and control group).

$H_a: \mu_e > \mu_k$ (the final geometry concepts comprehension of students who receive Van Hiele-Based Geometry Learning is better than that of students who receive Conventional Geometry Learning).

Table 4 shows that $F_{\text{calc}} = 83.346$, higher than $F_{\text{critical}} = 3.054$, on the significance level $\alpha = 0.05$, with degree of freedom two times 154 ($_{0.95}F_{2,154} = 3.054$); therefore, H_0 is rejected.

It is concluded that the final geometry concepts comprehension of the students who receive Van Hiele-Based Geometry Learning is better than that of the students who receive conventional geometry learning.

Analysis of interaction between the learning approach and school level on conceptual comprehension and mathematical communication skills is presented in Tables 4 and 5.

Table 5. Summary of Two-way ANOVA Test on Students' Final Geometry Concepts Comprehension based on School Levels and Learning Approaches

Source	JK	Dk	RJK	F _{hit}	F _{kritis}
School level (A)	603.019	2	301.509	6.703	
Learning (B)	3748.987	1	3748.987	83.346	3.05
A x B	207.284	2	103.642	2.304	

The results of the analysis in Tables 4 and 5 indicate that there is no interaction between learning and school level on conceptual comprehension and mathematical communication. The graphs of interaction are presented in Figures 3 and 4.

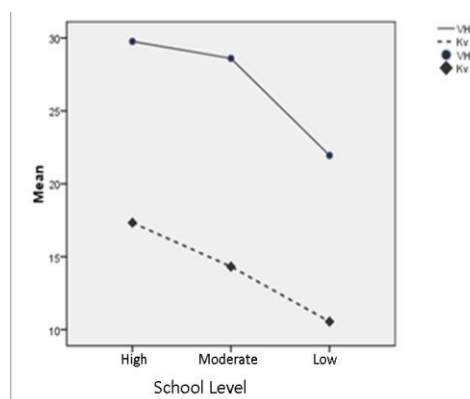


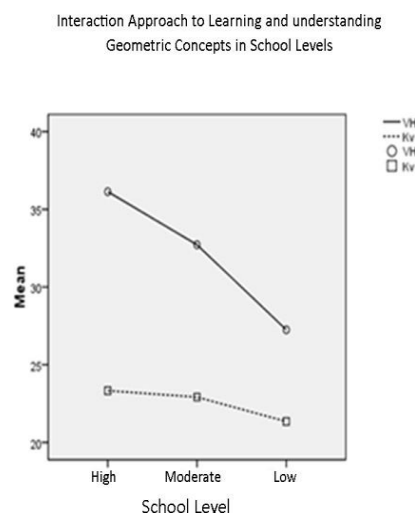
Figure 4. Interaction of Geometry Learning Approach and School Levels on the Geometric Conceptual Comprehension

D. Association between Students' Conceptual Comprehension Skill and Mathematical Communication Skill

To find out about the existence of association between students' conceptual comprehension skill and mathematical communication skill, the contingency association is used.

Table 6.
Scheffe Test of Mean Score of Students' Final Comprehension of Geometry Concepts
Based on School Level

Level (I)	Level (J)	Mean difference (I-J)	P	H ₀
High	Medium	1.74	0.396	Accepted
Medium	Low	2.72	0.141	Accepted
High	Low	4.46	0.004	Rejected



Based on the data in Table 6, p (0.004) is lower than 0.05 for the school levels of high and low, which means that H_0 is rejected. To summarize, there are two levels of schools (high and low) which are significantly different in terms of students' final comprehension of geometry concepts. Meanwhile, the school levels of high and medium and of medium and low are not significantly different in terms of students' final comprehension of geometry concepts.

The result of data analysis established that students' comprehension of geometric concepts through Van Hiele-Based Geometry Learning (VH) is better than that of students who learn through conventional learning (Kv).

The geometric concepts comprehension achievements of students who received Geometry Learning (VH and Kv) are in the category of medium; in which the average achievement is in the range of 17-32 of the Ideal Maximum score of 48.

The average score of geometric concepts comprehension achievement of students who receive geometry learning of VH is above 65% (67.31%), while the average score of comprehension achievement of students who receive Kv geometry learning is below 65% (47.23%).

This finding is congruent with Van Hiele's Theory [7], [8] that comprehension of geometric concepts is a skill developed through a process that builds from the previous schemes, previous understanding, and network of relationship among the concepts.

In Van Hiele-Based Geometry Learning, the students are motivated to be able to change the geometric concepts they already possess and understand, by comparing them with other concepts, so that they will have better comprehension of geometric concepts.

The finding of this study is also congruent with Piaget's theory that states that the most important part of comprehension is the evolutionary development of concepts; human continuously changes his ideas.

The first stage in the concept change is assimilation. In this stage, students use the concepts they already possess to face new concepts. However, at one point, the students will face new problem that cannot be solved with their old knowledge. At this point, they need to radically change their concepts. This stage is called accommodation.

The impact of Van Hiele-Based Geometry Learning on students' geometry comprehension is not inherent with other factors. The findings of this study show that there is no interaction between the Geometry Learning approaches (VH and Kv) with the school levels (H, M, L) in improving students' comprehension of geometry concepts.

It means that, altogether, the factors of geometry learning approaches and school levels do not provide significant contribution towards students' geometry comprehension achievement. This also means that the Van Hiele's theory is suitable for every student at every school level. In other words, the approach will not provide difficulties for students with low skills. Every asset of mental development and learning process are suitable for every student.

This study also shows that normalized gain for students' comprehension of geometry concepts in geometry learning based on the Van Hiele's theory is generally in the category of medium, while that of students' in conventional learning is in the category of low. It means that students' achievement of geometry comprehension is better when they are taught with VH learning than when they learn in Kv learning.

Conclusion

Based on data analysis and discussion, several conclusions can be drawn. Students' comprehension of geometry concepts in Van Hiele-Based Geometry Learning (VH); both in general and based on school level, is in the medium category, better than that of students' in conventional learning (Kv), which is in the low category. The superiority of Van Hiele-Based Geometry Learning is also supported by the finding that students' comprehension of geometry concepts in low level of VH learning is better than that of students' in medium and high level of Kv learning.

Students' level of geometric thinking in Van Hiele-Based Geometry Learning (VH) is higher than that of students' in conventional learning. Generally, students' level of geometric thinking in Van Hiele-Based Geometry Learning is on the High Visualization level, for all levels of schools (H, M, L). Some students are on the Analysis and Informal Deductive levels. Meanwhile, students' level of geometric thinking in conventional learning is generally on the Medium Visualization level; and some students are in the Low Visualization level.

Even though this research is not using a collaborative strategy, the students help each other in speaking, such as telling them the correct word in vocabulary or pronunciation. Not only that, but students also encourage their friends in the class so they can speak fluently and confidently when they are speaking English. This shows that the Spinning Wheel Game can make the students close to each other, responsible for each other, and also practice their speaking skills together in class. However, there are also disadvantage from Spinning Wheel Game. The disadvantages are that the teacher or student must have good equipment to make the teaching and learning process go smoothly, such as a laptop, smartphone, LCD projector, Wi-Fi, and a good signal. Another disadvantage is that due to the limited time and large number of students, some of them didn't get the chance to speak. Because of that, students who didn't have a chance to speak are bored and don't focus on the material.

References

- Bae, J., Bentler, P. M., & Lee, Y. (2016). On the Role of Content in Writing Assessment. *Language Assessment Quarterly*, 13(4), 302–328. <https://doi.org/10.1080/15434303.2016.1246552>.
- Bae, J., & Lee, Y. (2011). *Language Testing Assessment of Language Skills*. <https://doi.org/10.1177/0265532210382446>.
- Bitchener, J. (2012). Written Corrective Feedback for L2 Development: Current Knowledge and Future Research. *TESOL Quarterly*, 46(4), 855–860. <https://doi.org/10.1002/tesq.62>.
- Bitchener, J., & Knoch, U. (2008). The Value of Written Corrective Feedback for Migrant and International Students. *Language Teaching Research*, 12(3), 409–431. <https://doi.org/10.1177/1362168808089924>.
- Bitchener, J., Young, S., & Cameron, D. (2005a). The Effect of Different Types of Corrective Feedback on ESL Students' Writing. *Journal of Second Language Writing*, 14(3), 191–205. <https://doi.org/10.1016/j.jslw.2005.08.001>.
- Chang, W., Liao, C., & Chan, T. (2019). Improving Children Textual Cohesion and Writing Attitude in a Game-based Writing Environment. *Computer Assisted Language Learning*, 0(0), 1–26. <https://doi.org/10.1080/09588221.2019.1671459>.
- Chen, Y., Aguirre-mendez, C., Terada, T., Chen, Y., & Aguirre-mendez, C. (2020). Argumentative Writing as a Tool to Develop Conceptual and Epistemic Knowledge in a College Chemistry Course Designed for Non-science Majors. *International Journal of Science Education*, 0(0), 1–34. <https://doi.org/10.1080/09500693.2020.1837990>.
- DeKeyser, R. M. (1997). Beyond Explicit Rule Learning. *Studies in Second Language Acquisition*, 19(2), 195–221. <https://doi.org/10.1017/s0272263197002040>.
- Ellis, N. C. (2011). *Implicit and Explicit SLA and Their Interface*.
- Ellis, R., Sheen, Y., Murakami, M., & Takashima, H. (2008). The Effects of Focused and Unfocused Written Corrective Feedback in an English as a Foreign Language Context. *System*, 36(3), 353–371. <https://doi.org/10.1016/j.system.2008.02.001>.

- Evans, N. W., James Hartshorn, K., & Strong-Krause, D. (2011). The Efficacy of Dynamic Written Corrective Feedback for University-Matriculated ESL Learners. *System*, 39(2), 229–239. <https://doi.org/10.1016/j.system.2011.04.012>.
- Ferris, D., Brown, J., Liu, H., Eugenia, M., & Stine, A. (2011). Responding to L2 Students in College Writing Classes: Teacher Perspectives. *TESOL Quarterly*, 45(2), 207–234. <https://doi.org/10.5054/tq.2011.247706>.
- Gao, C. Z., Celce-Murcia, M., & Larsen-Freeman, D. (2000). The Grammar Book: An ESL/EFL Teacher's Course. *TESOL Quarterly*, 34(2), 372. <https://doi.org/10.2307/3587964>.
- Gharehbagh, M. J., Stapa, S. H., & Darus, S. (2019). *The Effects of Written Corrective Feedback Using Wikis among ESL Learners*. 25(1), 1–10.
- Hardman, W., & Bell, H. (2018). 'More Fronted Adverbials than Ever Before'. Writing Feedback Practices and Grammatical Metalanguage in an English Primary School. *Language and Education*, 0(0), 1–16. <https://doi.org/10.1080/09500782.2018.1488864>.
- Hosseiny, M. (2014). The Role of Direct and Indirect Written Corrective Feedback in Improving Iranian EFL Students' Writing Skill. *Procedia - Social and Behavioral Sciences*, 98, 668–674. <https://doi.org/10.1016/j.sbspro.2014.03.466>.
- Hsu, H., & Yuan, C. (2018). *Using Wiki-mediated Collaboration to Foster L2 Writing Performance*. 22(3), 103–123.
- Jacobs, G. M., Curtis, A., Braine, G., & Huang, S. Y. (1998). Feedback on Student Writing: Taking the Middle Path. *Journal of Second Language Writing*, 7(3), 307–317. [https://doi.org/10.1016/S1060-3743\(98\)90019-4](https://doi.org/10.1016/S1060-3743(98)90019-4).
- Kim, Y. J., Choi, B., Yun, H., Kim, B., & Choi, S. (2020). Task Repetition, Synchronous Written Corrective Feedback and The Learning of Korean Grammar: A Classroom-based Study. *Language Teaching Research*. <https://doi.org/10.1177/1362168820912354>.
- Kim, Youjin, & Choi, B. (2020). *Comparing the Effects of Direct and Indirect Synchronous Written Corrective Feedback: Learning Outcomes and Students' Perceptions*. January. <https://doi.org/10.1111/flan.12443>.
- Kim, Youjin, & Emeliyanova, L. (2019). *The Effects of Written Corrective Feedback on*

- The Accuracy of L2 Writing: Comparing Collaborative and Individual Revision Behavior.* <https://doi.org/10.1177/1362168819831406>.
- Kim, Youjin, & Kim, B. (2020). *Task Repetition, Synchronous Written Corrective Feedback and The Learning of Korean Grammar: A Classroom-based Study.* <https://doi.org/10.1177/1362168820912354>.
- Lee, I. (2020). Utility of Focused/Comprehensive Written Corrective Feedback Research for Authentic L2 Writing Classrooms. *Journal of Second Language Writing*, 49(May), 100734. <https://doi.org/10.1016/j.jslw.2020.100734>.
- Mei, W. S. (2001). *Investigating Raters' Use of Analytic Descriptors in Assessing Writing.* 9(2), 69–104.
- Nicolás–Conesa, F., Manchón, R. M., & Cerezo, L. (2019). The Effect of Unfocused Direct and Indirect Written Corrective Feedback on Rewritten Texts and New Texts: Looking into Feedback for Accuracy and Feedback for Acquisition. *Modern Language Journal*, 103(4), 848–873. <https://doi.org/10.1111/modl.12592>.
- Phillips, W. A., & Goodall, W. C. (2007). *Lexical Writing can be Non-semantic and It can be Fluent Without Practice.* March 2015, 37–41. <https://doi.org/10.1080/02643299508251994>.
- Rouhi, A., & Azizian, E. (2013). Peer Review: Is Giving Corrective Feedback Better than Receiving it in L2 Writing? *Procedia - Social and Behavioral Sciences*, 93, 1349–1354. <https://doi.org/10.1016/j.sbspro.2013.10.042>.
- Sarré, C., Grosbois, M., & Brudermann, C. (2019). Fostering Accuracy in L2 Writing: Impact of Different Types of Corrective Feedback in an Experimental Blended Learning EFL Course. *Computer Assisted Language Learning*. <https://doi.org/10.1080/09588221.2019.1635164>.
- Schmidt, R. W. (1990). The Role of Consciousness in Second Language Learning1. *Applied Linguistics*, 11(2), 129–158. <https://doi.org/10.1093/applin/11.2.129>.
- Sheen, Y. (2007). The Effect of Focused Written Corrective Feedback and Language Aptitude on ESL Learners' Acquisition of Articles. *TESOL Quarterly*, 41(2), 255–283. <https://doi.org/10.1002/j.1545-7249.2007.tb00059>.